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BIOGRAPHY.

TCHÉBYCHEV.

BY GEORGE BRUCE HALSTED.

PAFNUTI LVOVICH CHEBYSHEV in the notice which appeared in THE AMERICAN MATHEMATICAL MONTHLY, Vol. II., pp. 61-63, shortly after his death, was ranked as second only to Lobachévski among Russian mathematicians. In the memoir of Vasiliev which has just appeared the author says that if Russian mathematics may claim an honorable place in the modern science it is due to Lobachévski, to Ostrogradski, to Chebyshev.

The biography of Sylvester in the Proceedings of the Royal Society says of Sylvester's discourse on Conversion of Motion: "A synopsis only of the Royal Institution lecture was published. The manuscript of the lecture as actually delivered is in the possession of George Bruce Halsted, of the University of Texas. Extracts from it appear in the American Journal 'Science', of April 16, 1897, from which it appears that it was characterized by that eloquence, force, and poetical imagination with which students of Sylvester are familiar." From this precious unpublished MS. of the great "king of thought" I give the following: "My friend, Professor Tchebicheff, of the University of St. Petersburg (one of the greatest mathematicians of this or any previous age, the only man ever able to cope with the refractory character and erratic flow of prime numbers and to confine the stream of their progression within algebraic limits, building up, if I may so say, banks on either side which that stream, devious and irregular as are its windings, can never overflow), had long occupied himself with the theory of Parallel Motions. When he paid his first visit to me in England, at Lincoln's Inn Fields, about twenty years ago, his special object was to obtain sight of some of the old machines of Watt in order to ascertain if the parallel motions constructed by him followed strictly the description ordinarily given of them, or were adjusted in a manner to give more accurate results for a throw of the beam not indefinitely small, as the common theory virtually implies, but ranging within given limits.

I believe he found that the intuitive mechanical sagacity of Watt had anticipated his own mathematical deductions.

Since that time, among other researches of far greater importance, Tchebicheff has occupied himself with finding the best form of the 3-bar motion, of which Watt's parallel motion is only a special case, and arrived at results which give greater accuracy than Watt's at the same time that his arrangement is more compact, and, in some cases, more convenient of application. I may mention that in Tchebicheff's arrangement, the two radial bars cross instead of being parallel in the initial position ; and that while in Watt's arrangement the parallel point (*i. e.* the part meant to move straight) describes a figure of 8, in Tchebicheff's system it describes a very flat but open oval curve.

As regards the problem of a perfect parallel motion, Tchebicheff had satisfied himself that the solution was impossible, and had elaborated what seemed to him *almost* a conclusive proof that such a machine was inconstructible in the nature of things,—almost but not quite ; for under the very special hypothesis of two points usually distinct chancing to come into coincidence (if that could happen), he tells me that he foresaw that there was a bare possibility of his proof not holding water, and therefore with most praiseworthy caution, he held it back until he could succeed in patching up the supposed flaw. Still he made no secret of his inner conviction that the problem did not admit of solution ; nor did he stand alone among mathematicians of the highest rank in entertaining that belief.

On his recent visit (the third I believe which he has paid) to England, in the course of last autumn, he again called upon me, and naturally, knowing how much the subject had occupied his thoughts, I inquired after his old love, and asked him how fared his proof of the impossibility of a Perfect Parallel Motion. 'Oh,' replied my eminent guest, 'it is done !' 'Done !' said I, offering him my hand, 'I may then congratulate you on having at last overcome the difficulty with which you have so long contended.' 'No,' said he, with a somewhat downcast air, 'it has been done in France and again in Russia.' 'Somebody else, then, has obtained a proof of the problem being impossible.' 'No ; the problem has been solved, so that of course it cannot be demonstrated to be impossible.'

He then showed me Peaucellier's figure, which did indeed make me open my eyes, so simple—seeming to have so long eluded the gaze of the many sharp-sighted telescopic explorers watching to discover it flit over the wires ! 'Now, Dr. Tchebicheff,' said I, what do you think of the quadrature of the circle? Does not such a discovery shake our belief very much in what we call our intuitive convictions about things being possible or impossible?' 'My feeling,' was his answer, 'about the quadrature of the circle is this, that if the Emporer were to give me my choice to be shot at the expiration of a given time if I did not previously find out the quadrature, or else if I did not prove it to be impossible, I would elect to have to discover the quadrature.' [This was in 1874.]

He told me, too, how a pupil of his own, named Lipkin, a freshman at the University of St. Petersburg, had rediscovered Peaucellier's method in ignorance of what Peaucellier had done, and how at his own special recommendation, the Russian government had taken Lipkin in hand, and provided him with means to free himself from all anxiety for a livelihood, and to enable him to pursue his studies at the University with his mind at ease.

The air of Russia seems to no less favorable to mathematical acumen than to a genius for fable and song. Lobacheffsky, the first to mitigate the severity of the Euclidean code and to beat down the bars of a supposed adamantine necessity, was born (a Russian of Russians) in the government of Nijni Novgorod; Tchebicheff, the prince and conquerer of prime numbers, (who, rightly apprehending every fact as an actualized center, a ganglion of intellectual notions, seeks in realistic constructions and problems of every day life the source of his highest and subtlest mathematical inspirations), in the adjacent circumscription of Moscow, and our own Cayley, the central luminary of the mathematical firmament, was cradled amidst the snows of St. Petersburg.

I happened to receive a visit from M. Manuel Garcia immediately after Tchebicheff left me. I showed M. Garcia, who is a zealous geometer, the drawing of the cell and mounting of Peaucellier's instrument left by Tchebicheff. The next day to my surprise and gratification he brought me a working model of it in wood. When I took this instrument under my cloak and showed it after dinner to the Philosophical Club of the Royal Society it drew forth the most lively expressions of admiration from such men as Wheatstone. Tyndall, Sir C. Lyell, Dr. Carpenter, Mr. Busk, Sir B. Brodie, Mr. Playfair, and others. Soon after I exhibited the same model in the hall of the Athenaeum Club to my brilliant friend Sir William Thompson of Glasgow, who nursed it as if it had been his own child, and when a motion was made to relieve him of it replied: 'No! I have not had nearly enough of it—it is the most beautiful thing I have ever seen in my life.'

Had Peaucellier gone boldly to the Institute with his discovery, I feel fully persuaded from my personal knowledge of the men who adorn its lists, that he would not have had to wait nine years or to have depended on my accidental conference with Tchebicheff for his claims to have met with their first suitable recognition."

In a note, Sylvester says Tchebicheff told him he had succeeded in proving the non-existence of a five-bar linkwork capable of producing a perfect parallel motion. Peaucellier's required seven. But again the great Russian was mistaken ; for fired and inspired by this very lecture, Harry Hart went home and discovered the five-bar perfect parallel motion now known by his name, where for Peaucellier's cell of six bars is substituted Hart's contraparallelogram of four.

ON A METHOD TO CONSTRUCT INTRANSITIVE SUBSTITUTION GROUPS.

By DR. G. A. MILLER.

An intransitive group contains two or more transitive constituents and may be constructed by establishing some isomorphism between these constituent groups. If it contains more than two transitive constituents we may combine them in any way into two constituents and construct the group by establishing some isomorphism between these constituents, where at least one of them is an intransitive group. For example, we can find all the possible groups of degree 11 whose transitive constituents are of degrees 4, 4, 3 by establishing isomorphisms between the intransitive groups of degree 8 which contain two transitive constituents of degree 4 and the transitive groups of degree three. We can also find all these groups by establishing isomorphisms between the intransitive groups of degree 7 whose systems of intransitivity are 4, 3 and the transitive groups of degree 4.

Sometimes we can observe some important group properties from the notation by which an intransive group is represented. For instance, if the group is not simply isomorphic to some one of its transitive constituents it must contain at least two self-conjugate subgroups, differing from identity, that do not have any common operator besides identity. Conversely, if a group contains two such self-conjugate subgroups it may be represented as an intransitive group which is not simply isomorphic to any one of its transitive constituents.

While the method of establishing some isomorphism between two constituent groups seems to be the best general method to construct intransitive groups yet it is sometimes desirable to employ others. To illustrate one of these we shall employ it to find all the possible intransitive groups of degree 10 that contain five systems of intransitivity.

The average number of elements in all the substitutions of such a group is $10-5=5$[*] and a positive substitution must be of degree 4 or 8, while a negative substitution must be of degree 2, 6, or 10. There is evidently only one such group of order 2. If a group of order 4 contains only positive substitutions it must contain x substitutions of degree 4 and y of degree 8, where $x+y=3$. Hence

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[*] Frobenius, *Crelle*, vol. 101, p. 287; cf. Miller, *Bulletin of the American Mathematical Society*, vol. 2, 1895, p. 75.